Theory of Hindered Sedimentation of Polydisperse Mixtures

The theory and application of hindered sedimentation is reviewed. Polydisperse mixtures with size and density distributions are considered. A general solution technique is established for steady-flow regimes where a standard drag correlation for monodisperse systems is applied to each component. Solutions for discrete and continuous particle-size and density distributions are outlined. Results are presented for the sedimentation of mixtures made up of particles of uniform density and varying size suspended in a fluid with fixed properties. Under these conditions, the sedimentation velocities of the components vary significantly with the size distribution of the particles and the total volume fraction occupied by the particles In this context, topics such as maximum flux of particles, classification and density separation are considered.

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SCOPE

Hindred Sedimentation of particulate systems with different size and density distributions is considered in this study. The Settling of uniform particles has been a long-standing subject of research which produced many correlations for the estimation of terminal velocities as a function of the volumetric fraction ϕ occupied by the particles. A recent correlation produced by Barnea and Mizrahi is an advanced result of such intensive studies. Their results may be applied to uniform fractions settling in the Stokes, Intermediate, and Turbulent regimes, where reasonable accuracy is obtained in the complete range of practical values of ϕ, i.e., $0 < \phi < 0.7$. Batch sedimentation of fine particles was investigated by Richardson and Meikle in systems where the Richardson-Zaki equation was assumed to apply. Mirza and Richardson investigated mixtures uniform in density and of two or more sizes where again the same basic equation was used.

In both studies, a batch sedimentation model where different sedimentation zones developed was assumed to exist due to the presence of a number of sizes. The objective of this work is to provide a basic theory and calculation procedures of settling velocities of polydisperse mixtures by use of the new generalized theory of monodisperese systems. It applies to both discrete and continuous size and density distributions.

In this context determination of velocity distributions facilitates better understanding of the system and optimization of parameters such as flux, direction of sedimentation, efficiency of operation, etc.

Evaluation of sedimentation rates in batch and continuous systems is of theoretical and practical importance wherever there is a need for one discrete phase to move through another continuous phase. Such needs arise when increased interfacial areas are required for efficient mass and heat transfer and generally when the end product is produced as particulates in a wet process. Consequently, unit operations such as classification, thickening, clarification, dewatering, fluidization, solvent extraction, flotation, drying, cooling, and reactors involve sedimentation of mixtures to various degrees.

CONCLUSIONS AND SIGNIFICANCE

This study shows that the sedimentation velocities of different size and density fractions of a polydisperse mixture are sensitive not only to the total particle volumetric fraction but also to the size and density distributions. The steady sedimentation velocity of the ith fraction, of a polydisperse mixture comprising n fractions, can be evaluated by solution of a matrix equation derived from n equations with n unknown velocities defined with respect to the container. The known vector column comprises n values of velocities calculated with respect to the fluid using the generalized theory of monodisperse systems, the physical properties of each fraction, and the total volume occupied by the n fractions. It is hoped that the modified drag coefficient and the generalized theory facilitate better estimations of velocity distributions pertaining to practical polydisperse sedimentation systems.

The results indicate that smaller particles may move in a direction opposite to that of larger ones. This effect is enhanced at higher total concentrations. As concentration increases, the rising fluid is displaced by descending particles. It is expected to entrain larger particles and prevent them from moving in the desirable direction. On the other hand, such effect may be of value in special cases where it is beneficial to remove certain size fractions.

Optimized conditions for maximum flux (F) of particles, crossing a given level, depend on the properties of the mixtures, and the flow regime. Fine fractions that settle in the Stokes regime yield maximum flux at $\phi_{max} = 0.1692$, this value being independent of particle size. In this regime, the flux is the most sensitive to variation of the total concentration. Increase in particle size, i.e., transfer to the intermediate and turbulent regimes, increases ϕ_{max} and decreases the sensitivity to the total concentration. Hence, good control of concentration of fine particles slurries and thicker slurries of larger particles may be advantageous to enhance throughput.

In the case of mixtures, given the total concentration, the net flux increases with increase in particle-size difference. Furthermore, both the average particle size and standard deviation are characteristic variables of sedimentation systems and hence affect particle velocity distribution. This study relates to superior definitions of size and density separations with reference to the basic principles that underline these operations. Finally the theory and numerical example given in this study are significant for understanding processes that involve sedimentation of mixtures where classification, processing, measurement, handling, and general separation of particulates are required.

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INTRODUCTION

Hindered sedimentation (or flotation) of one phase in the form of particulates from another continuous phase is used in many chemical and mineral beneficiation processes. Typical examples are solvent extraction processes, fluidized beds, countercurrent reactors, classifications, thickening, density separation, and the jigging separation techniques.

A mixture of particulates in a sedimentation process may possess distributions of particle size, density, and shape, or a combination thereof. The hindrance effect in a mixture is more complex than the hindrance in a uniform system because the settling velocity of each component of the mixture is affected to different degrees by the counterflow of the displaced fluid due to the sedimentation of the other components. Additional complexity may be introduced by surface charges, adsorption and different degrees of aggregation, which are important in systems where physical or chemical flocculation enhance solid-liquid separation. Failure to understand the mechanism of the sedimentation of mixtures may result in undesirable variation from a predesigned separation process as well as problems of operation. For example, in classification operations where particles are separated according to size, a particular size fraction designed to settle may actually be swept up by the rising fluid displaced by other settling particles of the mixture.

In continuous solvent extraction processes, difficulties in phase separation may be due to similar reasons. Density separation (Perry and Chilton, 1973) as its name implies is believed to be independent of particle size. As will be shown later, this is true only when a dilute suspension and/or a uniform particle size are used. Dense slurries, with wide distributions of particle sizes and densities, are likely to show marked deviation from the designed separation characteristics. In such operations, small heavy particles may report to the light overflow fraction. Similar phenomena may occur in thickening operations where conventional settling calculations show the opposite, but unexplained operational problems such as sliming still occur.

In this paper nonflocculating polydispersed mixtures of spherical particles are considered. Procedures for estimation of sedimentation velocities of mixtures of particulates are outlined for discrete and continuous distributions of particle size. Calculation of particle and fluid fluxes, and optimization of sedimentation processes are studied. In this context, new interpretations for classification and density separation, and the meaning of equal settling (nonseparating) particles are discussed.

THEORY

Basic Assumptions

The theory of steady-state gravitational settling of particulate systems in Newtonian fluid has been reviewed by Barnea and Mizrahi (1973). They derived a general correlation for a system comprising uniform spherical particles with their settling velocity dependent on the volumetric fractions, the physical properties of the particles, and the fluid.

A fundamental assumption of the above theory is complete randomness of the system in the sense that particles and fluid are statistically uniformly dispersed at any time. Although local fluctuations that may lead to particle assemblies and collisions are possible, their effect is statistically assumed self-cancelling.

In cases of mixtures where all particles move in the same direction, probability of collisions increases because of differences in settling velocities of the various mixture components.

However, if no particle aggregation occurs, the effect of collisions is not expected to change significantly the basic assumption of randomness or the averaged terminal velocity or particle size of each component. An elastic collision between two particles is dampened by the fluid.

When in liquid, small particles attain their terminal velocity practically instantaneously. Hence, after recoil, such particles are immediately back at their terminal velocity. Once the colliding particles are separated they are back as part of the random system. Moreover, the probability of head-on collision and hence direct momentum transfer is small in dilute slurries, while the increased probability of collision in concentrated slurries is offset by the increased dampening effect.

In the following analysis, the effect of particle collision will be neglected. The hindrance effect is attributed to the mixture as a whole, and each component is driven by its own body force.

Sedimentation in Mixtures of Particles of Uniform Particle Size

The sedimentation characteristics of slurries can be studied by use of the terminal velocities proposed by Barnea and Mizrahi (1973). The terminal velocity of uniform spherical particles occupying a fractional volume ϕ can be evaluated by the use of the Dallavalle approximation (1948) as follows:

$$U_\phi = \left(\frac{-B + \sqrt{B^2 + AC}}{C}\right)^2 \qquad (1)$$

where

$$A = \alpha_1^{1/2} \qquad \alpha_1 = (\rho_s - \rho_f)(\rho_s + \rho_f)^{-1} g$$

$$B = \tfrac{1}{2}\alpha_2^{1/2}\alpha_4 \qquad \alpha_2 = (3/4)(1 + \phi^{1/3})[(\rho_s + \rho_f)d^2(1 - \phi)]^{-1}$$

$$C = \alpha_2^{1/2}\alpha_3 \qquad \alpha_3 = 0.63(d\rho_f)^{1/2}$$

$$\alpha_4 = 4.80\mu_f^{1/2}\exp\left(\frac{5}{6}\frac{\phi}{1-\phi}\right)$$

where U_ϕ is the relative velocity between the particles and the fluid. Equation 1 applies to the full sedimentation range, i.e., the Stokes, Transition, and Turbulent regimes. In the Stokes regime, Eq. 2 should be used.

$$U_\phi = \frac{(\rho_s - \rho_f)(1-\phi)gd^2}{18\mu_f A_\phi}, \quad A\phi = (1 + \phi^{1/3})\exp\left(\frac{5}{3}\frac{\phi}{1-\phi}\right) \qquad (2)$$

U_ϕ is related to the settling velocity by:

$$U_f = \phi U_d/(1 - \phi) \qquad (3)$$

$$U_\phi = U_d + U_f = U_d/(1-\phi) \qquad (4)$$

$$U_f = \phi U_\phi \qquad (4a)$$

where U_d is the velocity of particles relative to the container, and ϕ is the fractional volume occupied by the particles. Note that the term $U_f = U_d\phi/(1-\phi)$ represents the effect of flow of the fluid which is set by the settling particles. No external driving force other than the one associated with the motion of particles is involved.

Sedimentation in Mixtures of Particles of Different Sizes

For particles of the same type but of different sizes we have

$$\phi_t = \sum_{i=1}^{n} \phi_i \qquad (5)$$

where ϕ_i is the fractional volume occupied by component i of the n component mixture. The steady-state settling velocities of this mixture can be described by the following set of linear equations:

$$U_{\phi i} = \frac{1}{1-\phi_t}\sum_{j=1}^{m} U_{dj}\phi_j + U_{di}; \qquad i = 1,2,\ldots,n \qquad (6)$$

where the first term on the right hand is the upward velocity of the fluid (U_f) relative to the container.

$$U_f = \frac{1}{1-\phi_t}\sum_{j=1}^{n} U_{dj}\phi_j \qquad (7)$$

rearranging Eq. 6 and transforming it into a matrix form yields

$$\begin{pmatrix} \phi_1 + 1 - \phi_t & \phi_2 \ldots \phi_n \\ \phi_1 & \phi_2 + 1 - \phi_t \ldots \phi_n \\ \vdots & \vdots \qquad \vdots \\ \phi_1 & \phi_2 \ldots \qquad \phi_n + 1 - \phi_t \end{pmatrix} \times \begin{pmatrix} U_{dl} \\ U_{d2} \\ \vdots \\ U_{dn} \end{pmatrix} = (1 - \phi_t) \begin{pmatrix} U_{\phi 1} \\ U_{\phi 2} \\ \vdots \\ U_{\phi n} \end{pmatrix} \quad (8)$$

Equation 8 can be used for solving U_{di}, $i = 1,2,\ldots,n$ with values of $U_{\phi i}$, $i = 1,2,\ldots,n$ calculated from Eq. 1 or 2. $U_{\phi i}$ is calculated using the total fractional volume ϕ_t occupied by the mixture. The reasons for this are the following. By definition U_ϕ is a measure of the relative velocity between the particle and the fluid. The hindrance effect, as expressed in U_ϕ, is a function of ϕ (for example, Eqs. 1 and 2) and because of the randomness of the system is assumed to be independent of the motion of particles of a particular fraction ϕ_i. U_{di}, however, is dependent on the velocity of the fluid U_f according to Eq. 8, and hence on the composition of the mixture (Eq. 13). In Summary, the motion of the different components of the mixture does not change the hindrance characteristics of the system which is dependent on ϕ, and not on the manner it is distributed among the mixture components. This motion does change, however, the flow characteristics of the fluid and hence, of the particles, relative to the container. Following Eq. 8, care should be exercised in analysis and interpretation of results of classification, density separations, settling velocities in reactors and decanters, fluidized beds, and related unit operations. This is due to the effect of the size distribution on the vector column of U_d. For example, it is suggested that equal settling particles from different components of a mixture be defined by:

$$U_{di} = U_{dj} \qquad i \neq j \quad (9)$$

subject to Eq. 8. This subject and the case of mixtures comprising particle size as well as density distributions will be discussed later.

Continuous Particle-Size Distribution with Uniform Density

Consider a mixture with the following continuous particle-size distribution:

$$\phi = f(d) \quad (10)$$

with U_ϕ given by Eq. 4.

The velocity of the rising (displaced) water is,

$$U_f = \int_0^{\phi_t} \frac{U_d}{1 - \phi_t} d\phi, \text{ where } \phi_t = \int_0^{\phi_t} d\phi \quad (11)$$

but from Eq. 4. $U_d = U_\phi - U_f$, and since U_f and the total volume fraction ϕ_t are constant we have

$$U_f = \frac{1}{1 - \phi_t} \int_0^{\phi_t} (U_\phi - U_f) d\phi = \frac{1}{1 - \phi_t} \int_0^{\phi_t} U_\phi d\phi - \frac{\phi_t}{1 - \phi_t} U_f \quad (12)$$

hence

$$U_f = \int_0^{\phi_t} U_\phi d\phi \quad (13)$$

Using the definition of an average velocity, namely

$$\overline{U}_\phi = \frac{1}{\phi_t} \int_0^{\phi_t} U_\phi d\phi,$$

U_d can be expressed alternatively by:

$$U_d = U_\phi - \phi_t \overline{U}_\phi \quad (14)$$

Similar equations apply to discrete distributions.

The above analysis shows that U_d is a function of ϕ_t, d, ρ_s, ρ_f, μ_f and the characteristic particle-size distribution. In accordance with what has been already stated before, the critical condition $U_d = 0$ is of importance for design purposes of many processes that involve sedimentation systems. Particles with $U_d > 0$ will settle and those with $U_d < 0$ will be carried up by the fluid. This work shows that the counter flow characteristics of the particles and fluid depend on the particle-size distribution, on ϕ_t and on the physical properties of the two phases. The velocity of fluid can be evaluated for some useful particle-size distribution analytically in the Stokes regime, and numerically in the general flow regime. In the Stokes regime, combination of Eqs. 2 and 13 yields

$$U_f = \frac{(\rho_s - \rho_f)(1 - \phi_t)g}{18 \mu_f A_{\phi_t}} \int_{d=d_0}^{d=d_m \; \phi_t} [d(\phi)]^2 d\phi \quad (15)$$

Note that ϕ_t is constant.

Values of the integral for some useful particle size distributions are summarized in Table 1. Details of calculations are given in Appendix. For the general regime, combination of Eqs. 1 and 13 gives

$$U_f = \int_0^{\phi_t} \left(\frac{-B + \sqrt{B^2 + AC}}{C} \right)^2 d\phi \quad (16)$$

Evaluation of Flux Density

In this section monodisperse systems will be considered first, followed by polydisperse mixtures.

The flux density f, defined as volume of particles crossing a unit area per unit time, is given by:

$$f = \phi U_d = \phi(1 - \phi) U_\phi \quad (17)$$

which in the Stokes regime gives,

$$f = \frac{\phi(1 - \phi)^2(\rho_s - \rho_f) g d^2}{18 \mu_f A_\phi} \quad (18)$$

TABLE 1.
VALUES OF I* FOR SOME USEFUL SIZE DISTRIBUTIONS

Distribution	$\phi =$	$\int_{\phi=0, x=0}^{x=x(\phi_t), \phi=\phi_t} [d(\phi)]^2 d\phi$	Notes
Uniform	kx	$\frac{1}{3} k[x(\phi_t)]^3$,	$x(\phi_t)$ in finite.
Normal	$\frac{1}{\sqrt{2\pi}\sigma} \int_0^x e^{-(x-\alpha)^2/2\sigma^2} dx$,	$\frac{1}{2\sqrt{2\pi}} (\sqrt{2\pi}\, \sigma^2 + 4\alpha\sigma + \alpha^2 \sqrt{2\pi})$,	$x(\phi_t) = \infty$
Rosin Rammler	$\exp[-(x/x_0)^\alpha]$	$x_0^2 \left[t^{2/\alpha} e^{-t} - \frac{2}{\alpha} \int_0^t t^{2/\alpha - 1} e^{-t} dt \right]$,	$t = (x/x_0)^\alpha$ $x(\phi_t) = \infty$
Log Normal	$\frac{1}{\sqrt{2\pi}} \sigma_y \int_{-\infty}^x e^{-(y-\alpha_y)^2/2\sigma^2_y} dy$	$e^{2(\alpha_y + \sigma^2)}$,	$y = lnx$

* $I = \int_{\phi=0}^{\phi=\phi_t} [d(\phi)]^2 d\phi$

Maximum flux (Eq. 20) is obtained, from solution of $df/d\phi = 0$ at $\phi_{max} = 0.1692$ and is independent of particle size.

$$f_{max} = 0.3277\phi_{max}U_0 \tag{19}$$

U_0 is the Stokes free-settling terminal velocity. The factor 0.3277 is thus attributed to the hindrance effect at conditions of maximum flux.

Flux densities in the intermediate and turbulent regimes may be evaluated by combining Eqs. 1 and 17, and as shown later in this study ϕ_{max} and hence f_{max} become dependent on particle size. Consider a system in which the distribution characteristics of the particles is fixed independent of the total fractional volume, ϕ_t. In the following the relation between the particle flux and ϕ_t will be studied. The flux rate per unit area is given by:

$$F = \int_0^{\phi_b} U_d d\phi \tag{20}$$

where ϕ_b is arbitrary base value of the distribution. Consider a procedure for increasing ϕ_b by a factor α. In such a way, the distribution is kept fixed while the total fractional volume is increased. Subject to this condition Eq. 20 takes the form

$$F = \alpha \int_0^{\phi_b} [U_d(\alpha\phi)]d\phi = (1 - \alpha\phi_b)U_f$$
$$= (1 - \alpha\phi_b)\alpha \int_0^{\phi_b} [U_\phi(\alpha\phi)]d\phi \tag{21}$$

Equation 21 is appropriate for study and optimization of flux density in terms of its variation as a function of the total concentration parameter α, at fixed distribution.

Examples of flux calculations will be discussed in the sequel.

Sedimentation in Mixtures of Particles of Different Sizes and Densities

Consider m density fractions with each fraction comprising particle-size distribution as follows:

$$\phi_1 = \sum_{i1=1}^{n1-1} \phi_{i1}(d),\ \phi_2 = \sum_{i2=n1}^{n2-1} \phi_{i2}(d), \ldots, \phi_m = \sum_{im=n(m-1)}^{nm} \phi_{im}(d) \tag{22}$$

The densities are $\rho_1, \rho_2, \ldots, \rho_m$ for the $j = 1, 2, \ldots, m$ fraction respectively, and

$$\phi_t = \sum_{j=1}^{m} \phi_j.$$

The generalized matrix equation takes the following form:

$$\begin{pmatrix} \phi_{11} + 1 - \phi \ldots \phi_{n1-1}, \ldots, \phi_{n(m-1)} \cdots \phi_{nm} \\ \vdots \\ \phi_{11} \cdots \phi_{n1-1} + 1 - \phi, \ldots, \phi_{n(m-1)} \cdots \phi_{nm} \\ \vdots \\ \phi_{11} \cdots \phi_{n(1-1)}, \ldots, \phi_{n(m-1)} + 1 - \phi \ldots \phi_{nm} \\ \vdots \\ \phi_{11} \cdots \phi_{n1-1}, \ldots, \phi_{n(m-1)} \cdots \phi_{nm} + 1 - \phi \end{pmatrix} \times \begin{pmatrix} U_{d1}(\rho_1) \\ \vdots \\ U_{d(n1-1)}(\rho_1) \\ \vdots \\ U_{d\,n(m-1)}(\rho_m) \\ \vdots \\ U_{d\,nm}(\rho)_m \end{pmatrix} = \begin{pmatrix} U_{\phi1}(\rho_1) \\ \vdots \\ U_{\phi(n1-1)}(\rho_1) \\ \vdots \\ U\phi\, n(m-1)(\rho_m) \\ \vdots \\ U_{\phi nm}(\rho_m) \end{pmatrix} (1 - \phi_t) \tag{22a}$$

in accordance with the extended relation between $U_{\phi i}$ and U_{di}

$$U_{\phi i} = \frac{1}{1 - \phi_t} \sum_{j=1}^{nm} U_{dj}\phi_j + U_{di}, \quad i = 1,2,\ldots,nm \tag{22b}$$

Note that only the first and last groups of the density distribution, namely ρ_1 and ρ_m, are shown in the matrix equation, and the corresponding $U_d(\rho)$ and $U_\phi(\rho)$ are characterized by these densities (as well as other variables already mentioned). Application of the same theory to continuous distributions yields

$$U_f = \sum_{j=1}^{m} \int_0^{\phi_j} U_\phi(\rho_j,d,\phi_t)d\phi = \int_0^{\phi_t} U_\phi d\phi \tag{23}$$

The theory outlined in this section may be extended to nonsteady regimes, with the linear relations being replaced by symultaneous differential equations. This subject warrents a separate study.

Computer numerical solution of such equations can be done with initial values of $U_{\phi j} = 0$ and hence $U_f = \int_0^{\phi_t} U_\phi d\phi = 0$ at $t = 0$. Each step involves computation of $dU_{\phi j}$ leading to new values of $U_{\phi j}$ and the fluid velocity U_f. These values are used in turn in the next loop of computations. Values of F_{Dj} can be readily calculated from the following equations:
For the Stokes regime

$$F_{Dj} = 3\pi d\mu_f A_\phi U_{\phi j} \tag{24}$$

and for the intermediate and turbulent regimes

$$F_{Dj} = \frac{\pi}{8}\left[0.63(U_{\phi j}d\rho_f)^{1/2} + 4.80\mu_f^{1/2}\exp\left(\frac{5}{6}\frac{\phi_t}{1 - \phi_t}\right)\right] d(1 + \phi_t^{1/3})U_{\phi j} \tag{25}$$

Calculation of distances travelled by the particles is readily carried out by use of:

$$S_j = \int_0^t U_{dj}dt = \int_0^t (U_{\phi j} - U_f)dt = \int_0^t U_{\phi j}dt - \int_0^t \int_0^{\phi_t} U_\phi d\phi dt \tag{26}$$

Computations

For the purpose of illustrating the application of the theory of sedimentation to mixtures of spherical particles the normal size distribution was arbitrarily selected as:

$$\frac{d(\phi/\phi_t)}{d(d)} = \frac{1}{\sqrt{2\pi}\,\sigma} \exp[-(d - \bar{d})^2/2\sigma^2] \tag{27}$$

$\bar{d}$ is the average particle size and σ the standard deviation. The densities of the particles and fluid were fixed at 3.0 g/cm^3 and 1.0 g/cm^3 respectively and the fluid viscosity at 10^{-3} Pa·s. Calculations involved subdivision of the distribution into ten fractions with each fraction represented by its midpoint. For example, the particle size corresponding to $\phi/\phi_t = 0.35$ represents the fraction 0.30–0.40. Particle sizes corresponding to these particular midpoints were calculated as a function of the deviation from $\bar{d}$ in terms of units of standard deviations $X(\phi/\phi_t)$

$$d = \bar{d} + X(\phi/\phi_t).\ \sigma \tag{28}$$

$X(\phi/\phi_t)$ was obtained from standard mathematical tables. For $\phi/\phi_t = 0.05, 0.10, \ldots, 0.95$.

The variables studied were: $\bar{d}$, σ, and ϕ_t. Particle sizes smaller than 1 μm (including that part of the distribution with negative size values) were considered as uniform 1 μm fraction for all practical purposes. Equation 1 was used for evaluation of the right hand "U_ϕ column matrix" of Eq. 8 which in turn provided the solution for the U_d column matrix by use of a computer library routine.

RESULTS

Results related to velocity distributions are described first. Figure 1 shows a plot of U_d vs. D_p output for the series of values of ϕ_t studied where $\bar{d} = 0.01$ cm and $\sigma = 3\bar{d}$. The mixture comprised the following sizes: 0.0595, 0.0412, 0.0301, 0.0217, 0.0139, 0.0061, and 0.0001 cm, each representing a 10% fraction except for the last which is 40%. Two major effects of increase in ϕ_t on the U_d vs. D_p

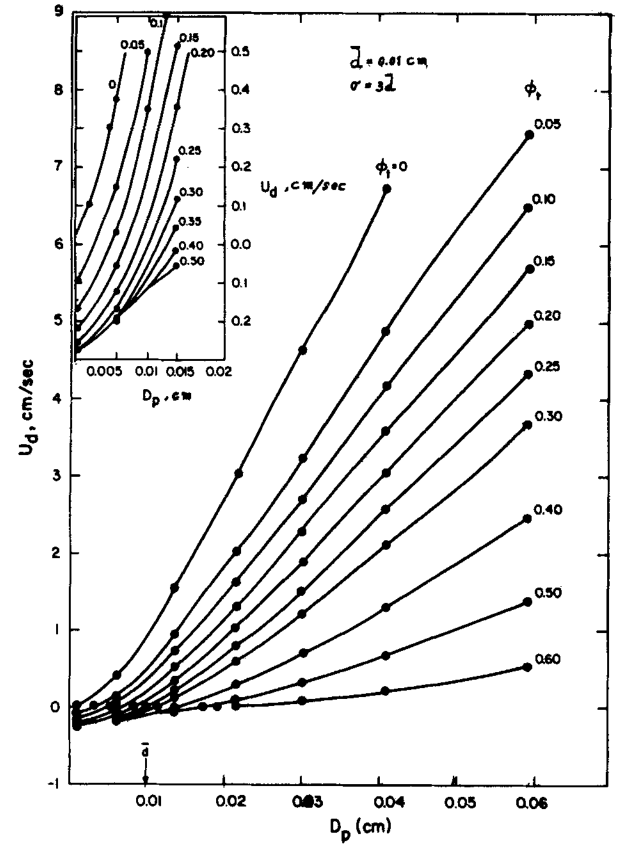

Figure 1. Results of calculations of U_d vs. D_p for different ϕ_t values. $\bar{d}$ = 0.01 cm, $\sigma = 3\bar{d}$.

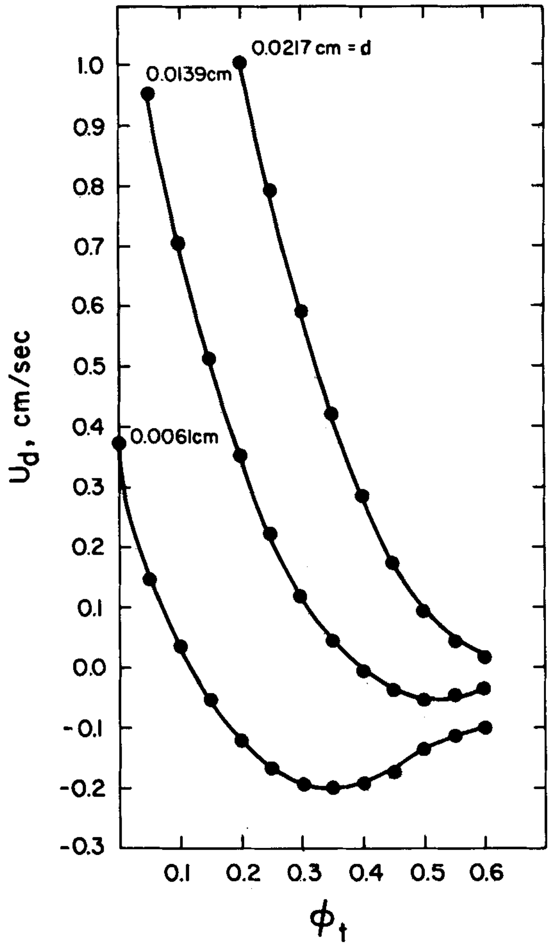

Figure 2. Plot of U_d vs. ϕ_t for three particle sizes in the mixture comprising a normal distribution. $\bar{d}$ = 0.0100 cm, $\sigma = 3.0\bar{d}$.

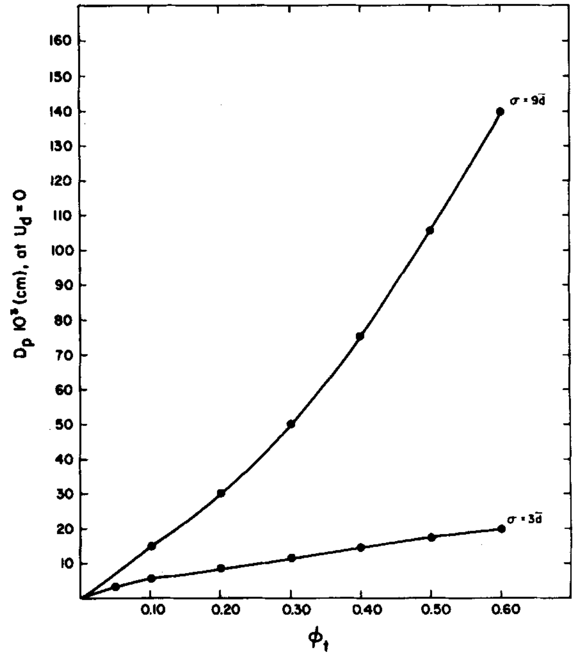

Figure 3. Critical values of particle size vs. ϕ_t that are stationary in the settling mixture. $\sigma = 3.0\bar{d}$ and $9.0\bar{d}$.

curves may be observed, namely a general decrease in U_d of the whole distribution and a stronger effect on smaller size fractions. As ϕ_t increases the rate of change $dU_d/d(D_p)$ decreases. Furthermore as shown by the insert, reversal of direction of motion of smaller particles may occur if a critical value of ϕ_t is exceeded.

Figure 2 shows a plot of U_d vs. ϕ_t for three particle sizes of the distribution. This type of plot shows the substantial effect of the larger particles on the motion of the smaller ones. The 0.0061-cm particles are slowed down considerably in the $\phi_t < 0.12$ range and reversal of direction occurs at $\phi_t = 0.12$. This hindrance is significant with regard to the expected flux of this size fraction. The minimum in the curve results from a trade off between increased fluid velocity and increased hindrance due to an increase in ϕ_t. For this particular distribution larger particles such as the 0.0217-cm size are too big to be carried up by the rising fluid. A basic principle in control of sedimentation processes may be inferred from the figure, i.e., reversal of direction of motion, given a specific distribution, can be controlled by suitable dilution.

Figure 3 shows a plot of critical values of particle size vs. ϕ_t that are stationary, i.e., $U_d = 0$, in the settling system for $\bar{d} = 0.01$ cm, and two standard deviations $\sigma = 3.0\,\bar{d}$, and $\sigma = 9.0\,\bar{d}$. Figure 3 was derived from plots such as Figure 1 and shows that the critical size of stationary particles increases with ϕ_t and with σ. The effect of increased standard deviation is rather dramatic particularly at large valued of ϕ_t, a three fold increase in σ from $3.0\bar{d}$ to $9.0\bar{d}$ increased significantly (more than threefold) the slope of the curve and the values of the critical sizes.

For example, at $\phi_t = 0.25$, the critical size is shifted from 100 μm at $\sigma = 3\bar{d}$ to 400 μm at $\sigma = 9\bar{d}$. It is seen that an increased standard deviation, i.e., spread of the distribution, is associated with an increase in critical sizes of the stationary particles.

The effect of an increase in average particle size on the properties of U_d vs. D_p relation is shown in Figure 4, where $\bar{d} = 0.1$ cm and $\sigma = 3.0\,\bar{d}$. The results (as compared to Figure 1) indicate that larger particles are less affected by the rising currents of the displaced fluid. Increase of $\bar{d}$ at a fixed σ decreases the relative effect of ϕ_t on U_d and on the rate of change of U_d with D_p. Furthermore, the rate of change of this slope decreases, with an increase in $\bar{d}$. Results of evaluation of D_p at $U_d = 0$ as a function of ϕ_t for $\bar{d} = 0.1$ cm and

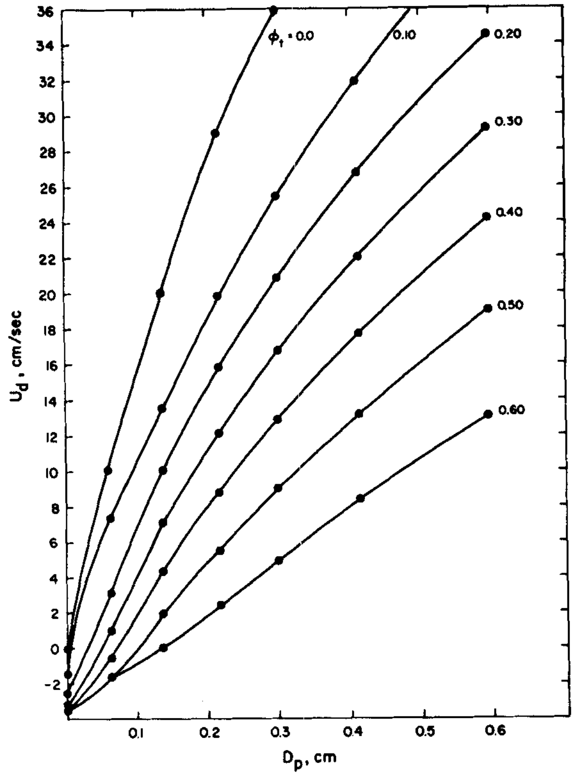

Figure 4. Results of calculations of U_d vs. D_p for different ϕ_t values $\bar{d}$ = 0.1 cm, σ = 3$\bar{d}$.

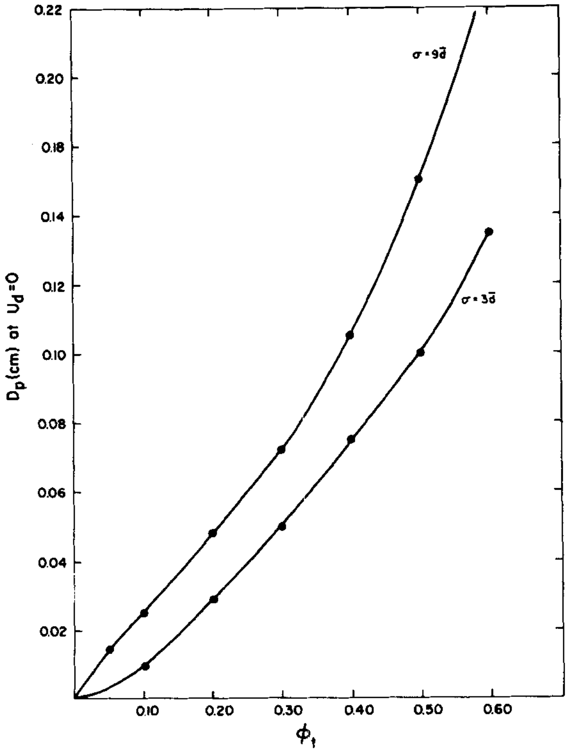

Figure 5. Critical values of particle size vs. ϕ_t that are stationary in the settling mixture. σ = 3.0$\bar{d}$ and 9.0$\bar{d}$.

two standard deviations $\sigma = 3.0\,\bar{d}$, $9.0\,\bar{d}$ are shown in Figure 5. Comparison with Figure 3 shows that the effect of σ on the critical sizes (at $U_d = 0$), for $0 < \phi_t < 0.6$, decreases with an increase in $\bar{d}$. Slopes and absolute values of $D_p(U_d = 0)$ vs. ϕ_t become closer for fractions of larger particle size. Results of calculations of flux density are described in the following:

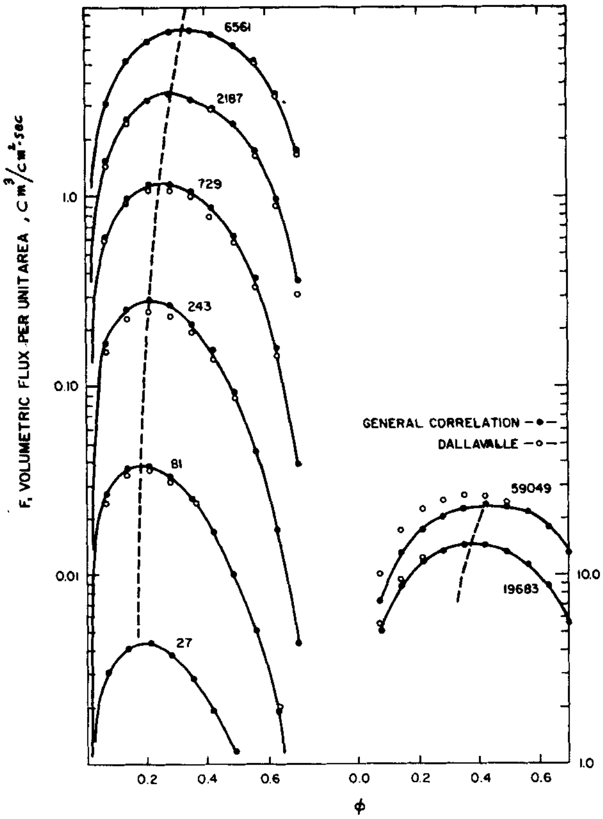

Figure 6. Results of calculations of flux density versus ϕ of monodisperse size fractions settling in water.

Figures 6 and 7 show results of calculations of flux density, as function of ϕ, of monodisperse size fractions settling in water and in air (at 18°C), respectively. In Figure 6 sizes are denoted on the curves in units of μm. The figures include data obtained by use of the Dallavalle approximation via Eqs. 1 and 18 and by direct use of the general correlation of Barnea and Mizrahi combined with Eq. 18. The following can be deduced regarding flux density of monodisperse fractions.

a. ϕ_{max} increases with particle size in the intermediate and turbulent regimes. The rate of change of ϕ_{max} with particle size increases from zero in the Stokes regime to a maximum in the turbulent regime.

b. The rate of change of $df/d\phi$ in the vicinity of the maximum flux density decreases with an increase of particle size.

c. The rate of change of maximum flux density with particle size decreases with an increase in particle size.

According to (a), denser suspensions are recommended for larger particle size. According to (b), the flux density of finer particles is more sensitive to changes in ϕ near ϕ_{max}. This does not apply at extreme conditions of dilution or concentrations. Figure 8 shows a plot of ϕ_{max} vs. d for monodisperse fractions settling in water and in air where the following logarithmic relation applies.

$$\log \phi_{max} = A_1 \log d + B_1, \; d > 50\mu m \tag{29}$$

$A_1 = 0.1309$, $B_1 = -0.9946$ for settling in water
$A_1 = 0.1320$, $B_1 = -0.9505$ for settling in air

As shown by the Figures 6 and 7 results calculated by use of the Dallavalle approximation are in good agreement with those obtained from the general corelation. Hence, use of the former is adequate for most practical purposes.

Figure 9 is a plot of particle flux density as a function of ϕ_t of two polydisperse mixtures defined by the normal distribution with $\bar{d}$ = 0.010, 0.100 cm, and $\sigma = 3.0\,\bar{d}$, $9.0\,\bar{d}$, respectively. Increase in $\bar{d}$ and in σ shifts the maximum flux to higher values of ϕ_t. For this particular example the maxima are rather flat, hence high flux

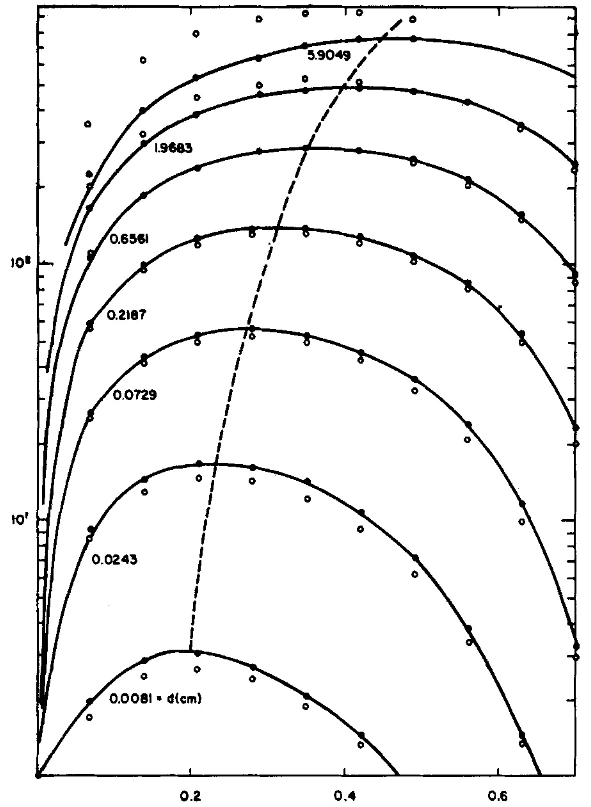

Figure 7. Results of calculations of flux density vs. ϕ of monodisperse size fractions settling in air at 18°C, 1at.

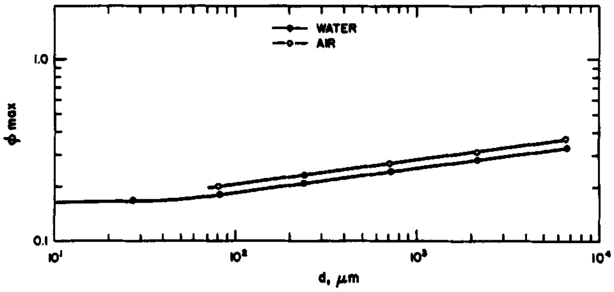

Figure 8. A plot of log ϕ_{max} vs. log d.

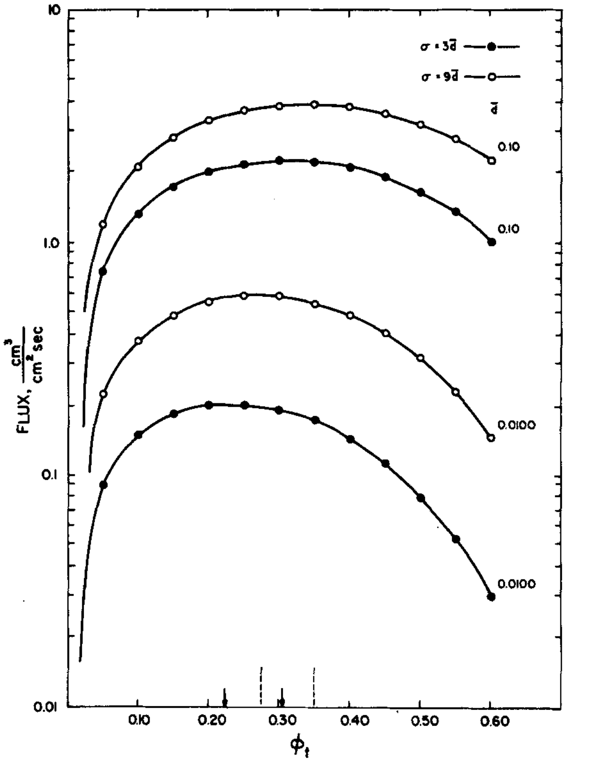

Figure 9. Results of flux density vs. ϕ_t, for $\bar{d}$ = 0.01cm, 0.1 cm and $\sigma = 3.0\bar{d}$ and $9.0\bar{d}$.

densities can be maintained over a wide range of ϕ_t, for example, the 1.0-cm fraction. The effect of σ is similar to the one in monodisperse fractions. Increase in σ results in increase in flux, but for larger values of $\bar{d}$ to a lesser degree. For polydisperse fractions with similar distributions it is expected that flux densities will attain values reasonably close to maximum in the $0.16 \leq \phi \leq 0.26$ range.

DISCUSSION

The results of the above theory of sedimentation concerning mono- and polydisperse mixtures apply to real dispersed systems of particulates. Whenever the basic properties of a mixture can be defined in terms of particle size and density, its sedimentation behavior may be estimated both in trend and in absolute values. Determination of velocity distribution yields upon further computations data on efficiency of operation as compared to design goals. The basic assumptions of dispersed spherical particles where interparticle collisions are neglected are valid in cases where general trends are studied. They are also supported by the well-known segregation that takes place in a batch sedimentation process (Mirza and Richardson, 1979). It seems that the rising currents overwhelms the effect of collisions in particular if they are perfectly elastic. Increased deviation is expected in systems that involve plastic collisions. The extent to which collisions effect the above theory needs more work; however, this is outside the scope of this study. In the following reference is made to processes that involve sedimentation of polydisperse mixtures.

Sedimentation is involved to various degrees of importance in processes comprising two (or more) phase flow. For example, transportation and agitation of slurries depend on prevention of settling of the suspended solids. Classification, fluidization and elutriation operations are designed to meet the sedimentation characteristics of the particulates. Phase separation in solvent extraction depends on the distribution of the dispersed phase in the mixer settler. Thickening and centrifugation are controlled by sedimentation. Design of reactors that utilize countercurrent flow of phases involve consideration of sedimentation. Density separation involves separation of the feed into two streams moving in opposite directions—the heavy particles to the underflow (sink) and the light one to the overflow (float). These streams are controlled by the laws of sedimentation. Areation of liquids, for the purpose of flotation, purification or fermentation, can be included in this category as well. The ramification of the theory of sedimentation of mixtures, as viewed from specific application, is considered next.

Classification

Classification, by definition, is a unit operation whose purpose is to separate a mixture of particles into well-defined size fractions.

Fine mixtures are generally classified by wet processes using sedimentation either in gravitational or centrifugal fields. Dilute slurries are advantageous because of reduced hindrance and particle interactions. However, this increases the demand for water and energy to handle the required capacity. The capacity of wet classifiers is proportional to the flux of particles through the classification zone. As illustrated by Figures 6, 7 and 9, maximum flux can be obtained in slurries denser than $\phi_t = 0.163$. Design of such classification should involve the principles and theory presented in this work.

Phase Separation in Extraction Processes

Phase separation between the dispersed and the continuous phase involves a process of coalescence of drops and sedimentation. In batch operation the distribution of the drops and hence their sedimentation velocities are time-variable. The larger, faster drops displace the small ones. In steady continuous processes the drop distribution is time-invariant, and small drops may be lost continuously depending on the particular distribution used. This drop entrainment may be estimated by use of a modified version of the theory presented here. This, however, is outside the scope of this work.

Density Separation

Density separation by definition is a unit operation for the purpose of separation of particulates into fractions with well-defined density range. It is implied by this definition that the separation should be independent of variables such as particle shape and size. Conditions for true density separation exist when the size and shape of the particulates are uniform or when free settling prevails, i.e., in dilute slurries. Density separation is dependent on the distribution characteristics in all other cases, and small, heavy particles may report to the light fraction. Studies of density separation may be done by use of the theory of sedimentation of mixtures.

NOTATION

A = $\alpha_1^{1/2}$
A_1 = constant
A_ϕ = $(1 + \phi^{1/3}) \exp\left(\frac{5}{3}\frac{\phi}{1-\phi}\right)$
B = $\frac{1}{2}\alpha_2^{1/2}\alpha_4$
B_1 = constant
C = $\alpha_2^{1/2}\alpha_3$
d = particle size; also subsindex denoting velocity with reference to the container and symbole of differentiation
$\bar{d}$ = average particle size of a size distribution
D_p = particle diameter
f = flux density; also used as subindex denoting a value pertaining to the fluid and as a symbol of a function
F = integrated flux density
F_D = drag force
g = gravity acceleration
i,j,m,n = subindices denoting the ith, jth, mth, and nth term, respectively
S = distance
s = subindex denoting value pertaining to the particulate
t = subindex denoting a total quantity, also used as an integration variable
U_d,U_f = velocities relative to the container of particulate, and fluid, respectively
U_ϕ = average velocity of a particle relative to the fluid
U_0 = terminal free settling velocity in the Stokes regime
X = units of standard deviation

Greek Symbols

α = total concentration parameter for a fixed distribution
α_1 = $(\rho_s - \rho_f)(\rho_s + \rho_f)^{-1}$
α_2 = $\left(\frac{3}{4}\right)(1 + \phi^{1/3})(\rho_s + \rho_f)d^2(1-\phi)^{-1}$
α_3 = $0.63(d\rho_f)^{1/2}$
α_4 = $4.80\mu_f^{1/2} \exp\left(\frac{5}{6}\frac{\phi}{1-\phi}\right)$
μ_f = viscosity of the fluid
ρ_f,ρ_s = densities of fluid, and particulates, respectively
σ = standard deviation
ϕ = fractional volume occupied by particulates; also used as subindex to denote a quantity dependent on the volume fraction
ϕ_i,ϕ_j,ϕ_n = fractional volume of the ith, jth, and nth fraction respectively
ϕ_t = total fractional volume
ϕ_{max} = fractional volume at maximum flux density

APPENDIX

The normal distribution is given by

$$d\phi = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\alpha)^2/2\sigma^2}dx$$

where x is the particle size at ϕ, α is the average particle size and σ the standard deviation. The range of integration for particle-size distributions is $0 \le x < \infty$. Therefore, this distribution can be used only in a "truncated" form where the $-\infty < x < 0$ range is not included. If α is not too large this range is expected to be negligible.

$$I = \int_0^\infty x^2 d\phi = \int_0^\infty x^2 \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\alpha)^2/2\sigma^2}dx$$

$$t = x - \alpha, \qquad dx = dt, \qquad x^2 = (t+\alpha)^2$$

$$I = \frac{1}{\sqrt{2\pi}\sigma}\int_0^\infty (t+\alpha)^2 e^{-t^2/2\sigma^2}dt$$

$$= \frac{1}{\sqrt{2\pi}\sigma}\left[\int_0^\infty t^2 e^{-t^2/2\sigma^2}dt + 2\alpha\int_0^\infty t e^{-t^2/2\sigma^2}dt + \alpha^2\int_0^\infty e^{-t^2/2\sigma^2}dt\right]$$

$$= \frac{1}{\sqrt{2\pi}\sigma}\left[(2\sigma^3)^{3/2}\frac{\sqrt{\pi}}{4} + 2\alpha 2\sigma^2\frac{1}{2} + \alpha^2\frac{1}{2\frac{1}{\sqrt{2\sigma^2}}}\sqrt{\pi}\right] = \frac{1}{2\sqrt{2\pi}}[\sqrt{2\pi}\sigma^2 + 4\alpha\sigma + \alpha^2\sqrt{2\pi}]$$

Similar calculations apply to the Log-Normal distribution.

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